APPROXIMATE CALCULATION OF THE ADDED MASS IN FLOW AROUND A MULTIROW ARRAY OF RODS

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A method of calculating effective masses of multirow clusters of elastic-cylindrical rods is presented. The flow of liquid caused by lateral vibrations of the rods is described approximately using a model of cells.

1. The Model of the Cells. The work [1] is devoted to calculation of effective masses of single-row clusters of cylindrical rods. As far as calculation of effective masses of multirow clusters is concerned, this has been very little studied up to now because of the complexity of taking into account the restriction of the flow of liquid in the cluster.

The proposed method is based on replacing the multirow cluster of cylindrical rods (Fig. 1a) by a combination of isolated cells, one of which is shown in Fig. 1b. The cell consists of two coaxial cylinders; the inner cylinder represents a rod of the cluster, and the outer cylinder simulates the restriction of the flow which is caused by lateral vibrations of the inner cylinder.

Let each rod of the cluster be surrounded by a liquid which fills a certain region (Fig. 1c). It is then possible to find from the condition of continuity of flow of liquid flowing through the cell and through the above mentioned region a relationship between the radius of the outer cylinder b and the spacing h of the rods in the cluster, and also the connection between the relationship between the radii of the cylinders forming the cells and the density of the cluster

$$b = \pi^{-1/2} h, \qquad a / b = {}^1/_2 \pi^{1/2} q \qquad (q = 2a / h) \tag{1.1}$$

where a is the outer radius of the rod of the cluster, and q is the density of the cluster.

Hence, determination of the effective mass of a rod located in the cluster is reduced to calculation of the effective mass of this rod located inside a cylinder with a radius b.

The problem is solved by the hypothesis that the rods of the cluster have a finite length and arbitrary supports at the ends, and they carry out small elastic vibrations.

The liquid which flows round the rod is considered as ideal and compressible, but its flow is considered as irrotational.

The flow of liquid in the cell is described by the wave equation

$$\frac{\partial^2\varphi}{\partial r^2} + \frac{1}{r}\frac{\partial\varphi}{\partial r} + \frac{1}{r^2}\frac{\partial^2\varphi}{\partial\theta^2} + \frac{\partial^2\varphi}{\partial z^2} - \frac{1}{c}\frac{\partial^2\varphi}{\partial t^2} = 0 \tag{1.2}$$

Leningrad. Translated from Zhurnal Prikladnoi Mekhaniki i Tekhnicheskoi Fiziki, No. 5, pp. 50-54, September-October, 1970. Original article submitted December 23, 1969.

with the following boundary conditions:

$$\left.\frac{\partial\varphi}{\partial r}\right|_{r=a}=\frac{\partial x}{\partial t}\cos\theta,\quad \left.\frac{\partial\varphi}{\partial r}\right|_{r=b}=0$$
$$\left.\frac{\partial\varphi}{\partial z}\right|_{z=0}=0,\qquad \left.\frac{\partial\varphi}{\partial z}\right|_{z=l}=0 \tag{1.3}$$

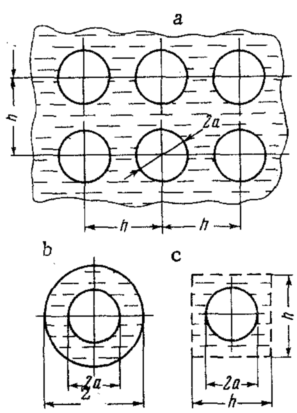

Fig. 1

Here φ is the velocity potential, l is the length of the rod, and x is the displacement of the rod which is equal to

$$x=\sum_{n=1}^{\infty}q_n(t)f_n(z),\qquad q_n=C_1\cos p_n{}^\circ t+C_2\sin p_n{}^\circ t \tag{1.4}$$

In these expressions f_n is the form of the n-th mode of the vibrations of the rod in the liquid, which has been taken to be equal to the mode of the vibrations in a vacuum; q_n is the main coordinate; C_1 and C_2 are the arbitrary constants determined by the initial conditions of the problem; $p_n{}^\circ$ is the frequency of the n-th mode of the free vibrations of the rod in the liquid.

The velocity potential is determined in the following form:

$$\varphi=\cos\theta\sum_{n=1}^{\infty}q_n{}^{\cdot}\sum_{s=1}^{\infty}F_s(r)N_s(z) \tag{1.5}$$

The function $F_s(r)$ must satisfy the first two boundary conditions (1.3), and the function $N_s(z)$ must satisfy the remaining boundary conditions. The two latter conditions are satisfied if we assume

$$N_s(z)=\cos(\pi sz\,/\,l) \tag{1.6}$$

By substituting the expression (1.5) into the original Eqs. (1.2), taking (1.6) into account, we have

$$\frac{d^2F_s}{dr^2}+\frac{1}{r}\frac{dF_s}{dr}-\left[\left(\frac{\pi s}{l}\right)^2+\frac{1}{r}\left(\frac{p_n{}^\circ}{c}\right)^2\right]F_s=0 \tag{1.7}$$

The solution of this equation will be

$$F_s=AI_1\left[\left(\frac{\pi s}{l}+\frac{p_n{}^\circ}{c}\right)r\right]+BK_1\left[\left(\frac{\pi s}{l}+\frac{p_n{}^\circ}{c}\right)r\right] \tag{1.8}$$

Here A and B are arbitrary constants; I_1 and K_1 are modified Bessel functions.

After calculation of the arbitrary constants [using the first two conditions of (1.3)], the formula for the velocity potential assumes the form

$$\varphi=\frac{2}{\pi}\cos\theta\sum_{n=1}^{\infty}q_n{}^{\cdot}\sum_{s=1}^{\infty}\left[K_1(\alpha r)-\frac{K_1{}^{\cdot}(\alpha b)}{I_1{}^{\cdot}(\alpha b)}I_1(\alpha r)\right]$$
$$\times\left[K_1{}^{\cdot}(\alpha a)-\frac{K_1{}^{\cdot}(\alpha b)}{I_1{}^{\cdot}(\alpha b)}I_1{}^{\cdot}(\alpha a)\right]^{-1}\frac{1}{s}\cos\frac{\pi sz}{l}\int_0^l f_n(z)\cos\frac{\pi sz}{l}\,dz\qquad\left(\alpha=\frac{\pi s}{l}+\frac{p_n{}^\circ}{c}\right) \tag{1.9}$$

The kinetic energy of the liquid which fills the doubly connected region is equal to

$$T=-\frac{\rho^\circ}{2}\iint_{S_1}\varphi\frac{\partial\varphi}{\partial n}dS_1 \tag{1.10}$$

Here S_1 is the surface of the rod, and ρ° is the density of the liquid.

A similar integral over the surface of the outer cylinder of the cell in the same formula disappears due to the second condition (1.3).

After substitution of the value of the velocity potential into this formula and integration, which is carried out over the surface of the rod, the kinetic energy of the liquid is determined

$$T=a\rho^\circ\sum_{n=1}^{\infty}q_n{}^{\cdot 2}\sum_{s=1}^{\infty}\frac{\xi_{sn}}{s}\left[\int_0^l f_n(z)\cos\frac{\pi sz}{l}\,dz\right]^2 \tag{1.11}$$

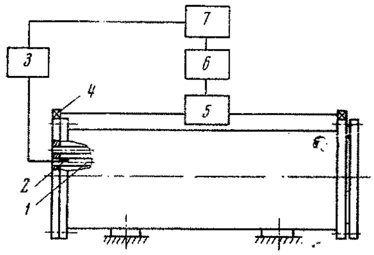

Fig. 2

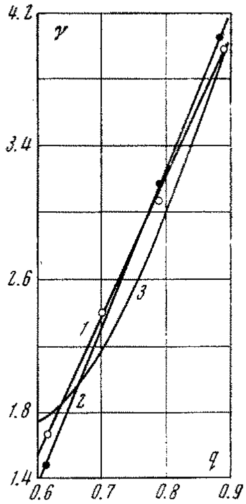

Fig. 3

In this equation

$$\xi_{sn} = \frac{K_1(\beta_1) + I_1(\beta_1)\zeta(\beta_2)}{K_0(\beta_1) + \beta_1^{-1}K_1(\beta_1) - [I_0(\beta_1) - \beta_1^{-1}I_1(\beta_1)]\zeta(\beta_2)}$$
$$\beta_1 = \left(\frac{\pi s}{l} + \frac{p_n^\circ}{c}\right)a, \quad \beta_2 = \frac{\beta_1 b}{a}, \quad \zeta(\beta_2) = \frac{\beta_2 K_0(\beta_2) + K_1(\beta_2)}{\beta_2 I_0(\beta_2) - I_1(\beta_2)} \tag{1.12}$$

For kinetic energy of the liquid we have [2]

$$T = {}^1/_2 \sum_{n=1}^{\infty} M_n^\circ \dot{q}_n^2 \tag{1.13}$$

Here M_n° is the reduced effective mass of the rod which is located in the cell; this connected mass corresponds with the n-th mode of vibration; q_n is the velocity of the reduced point of the rod when it vibrates in the n-th mode.

If we compare the right-hand part of Eqs. (1.11) and (1.13), we have

$$M_n^\circ = 2\frac{m_0}{\pi a}\sum_{s=1}^{\infty}\frac{\xi_{sn}}{s}\left[\int_0^l f_n(z)\cos\frac{\pi s z}{l}\,dz\right]^2 \tag{1.14}$$

2. Experimental Basis of the Model of the Cells. When a cluster is replaced by a combination of isolated cells in a calculation of effective masses, a certain error is introduced. Firstly, in the case of a substitution of this kind the arrangement of the rods in the cluster (straight, staggered, etc.) is not taken into account. Secondly, the actual clusters consist of a finite number of rods, and therefore the flow round the central and peripheral rods is different.

The influence of the above-mentioned factors on the connected mass of clusters was analyzed using experimental data obtained on an experimental apparatus; a diagram is given in Fig. 2.

A cluster of tubes 1 was used as the model of the cluster. The space between the tubes was filled with water.

The vibrations of the tubes were excited by a momentary pulse which was generated by the impact of the plate of the impact machine, on which the model is fixed, against the shock absorber.

Recording of the vibrational process is carried out by a strain-gauge circuit which consists of a strain gauge 2 and a converter 3.

The boundary of the area of free vibrations of the tubes was established from the recordings of the accelerations of their support sections; these were carried out by the measuring unit which consisted of acceleration transducers 4, a cathode follower 5, and an amplifier 6. The processes were recorded by a loop oscillograph 7.

The coefficients of the effective masses were calculated according to the formula

$$\gamma = \frac{M^\circ}{M_0} = \frac{M}{M_0}\left[\left(\frac{p}{p^\circ}\right)^2 - 1\right] \tag{2.1}$$

Here M° is the effective mass of the tubes; M is the mass of the tubes; M_0 is the mass of water enclosed by the tubes; p, p° are the frequencies of the free vibrations of the tubes in air and water, determined according to the oscillograms of the stresses.

The tests were carried out at cluster densities of 0.61, 0.70, 0.78, and 0.87.

In order to evaluate the influence of the form of layout of the rods in the clusters on the magnitude of the effective masses, clusters with staggered and straight arrangement of the tubes were tested. The tests showed that the form arrangement does not have a substantial influence on the effective mass in the case of the same densities of the clusters (Fig. 3, curve 1 shows a straight arrangement of the tubes; curve 2 shows a staggered arrangement of the tubes).

By determining the coefficients γ for tubes located in different parts of the cluster, the problem of the difference in the magnitudes of the effective masses of the central and peripheral tubes is explained. The test results showed that this difference for the same cluster did not exceed 10-15%.

This means that for a multirow cluster containing k rods the effective mass can be determined as the effective mass of one rod, which is multiplied k times.

3. Analysis of the Results Obtained. It is found from (1.14) that the effective mass of the rod, which is located in the cell, depends on the dimensionless magnitude $p_n^\circ a/c$, which characterizes the compressibility of the liquid. Analysis shows that the reduction of the effective mass by the compressibility of the liquid only becomes noticeable in the case of high-frequency vibrations ($p_n^\circ a/c > 2$). In practical calculations, therefore, the liquid can be considered as incompressible, i.e., in the ratio (1.12) it is assumed that $c = \infty$.

Taking the above into account, the expression for determining the effective mass of the clusters of rods has the form

$$M_n^\circ = 2k\frac{m_0}{\pi a}\sum_{s=1}^{\infty}\frac{\xi_{sn}}{s}\left[\int_0^l f_n(z)\cos\frac{\pi s z}{l}dz\right]^2 \tag{3.1}$$

where the coefficient ξ_{sn} is calculated according to Eq. (1.12) for the values

$$\beta_1 = \pi s a / l, \ \beta_2 = 2\pi^{-1/2}\beta_1 / q \tag{3.2}$$

The series in Eq. (3.1) converges rapidly. In many cases the first two or three terms of the series give an accuracy which is sufficient for practical purposes.

It is seen from Eq. (3.2) that the effective mass of the cluster depends on the relative length of the rods l/a and the density q.

For relatively short rods ($l/a < 5$) the axial component of the movement of the particles of the liquid has a considerable magnitude. With increase in the relative length, however, the part played by this component decreases, and when $l/a \geq 30$ it is possible to take into account only the radial component of the movement of the particles of the liquid, that is, the flow is considered to be plane.

With regard to the density of the cluster, as this increases the effective mass increases, which is seen from the graph of the relationship $\gamma(q)$, which is given in Fig. 3 (curve 3) and constructed for the case $l/a = 30$.

When this graph is compared with curves 1 and 2, which are plotted according to experimental data, it is seen that the calculated data correspond sufficiently well with the results of the tests. This means that it is possible to use the model which we have examined for approximate calculations of effective masses of multirow clusters of rods.

LITERATURE CITED

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2. Yu. A. Shimanskii, Dynamic Calculation of Marine Constructions [in Russian], Sudpromgiz, Leningrad (1963).